

Frictional Resistance in Artificially Roughened Pipes

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VICTOR L. STREETER

Ann Arbor, Michigan

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FRICITIONAL RESISTANCE IN ARTIFICIALLY ROUGHENED PIPES

BY VICTOR L. STREETER,¹ JUN. AM. SOC. C. E.

SYNOPSIS

The flow of fluids in smooth pipes is so well understood that the losses due to friction may be predicted to within 5 per cent. The problem of estimating friction losses in old pipes, or in those roughened by use, however, has been inadequately solved. In order to clarify the problem completely much study and experimentation are necessary. This paper, which is an effort in that direction, presents the results of an experimental investigation of frictional resistance in artificially roughened pipes. It has been undertaken in an effort to show, qualitatively, the effect on the friction factor of certain artificial irregularities—varying in shape and size—that were introduced in pipes used for the tests. With water as the fluid, loss of head was investigated for roughened, 2-in., brass pipe, the Reynolds numbers ranging from 20 000 to 1 250 000. The roughness elements consisted of grooves cut spirally into the pipe. The degree of roughness was varied by changing the depth, shape, and number of grooves per inch.

The loss of head due to friction was found to increase with an increase in the depth of the grooves, provided the same general shape was maintained. The shape of the grooves, however, seems to have almost as much affect on the loss of head as the depth. The experiments also indicate that the roughness elements may be placed so close together that the loss of head is reduced. The work of previous investigators is summarized briefly; the apparatus, preliminary experiments, experiments for obtaining data, and computations are described; and the results of this investigation are discussed.

NOTATION

The symbols in this paper are introduced in the text as they occur and are summarized for reference in Appendix I. An effort has been made to

NOTE.—Discussion on this paper will be closed in May, 1935, *Proceedings*.

¹ Junior Engr., U. S. Bureau of Reclamation, Denver, Colo.

conform essentially with "Symbols for Hydraulics,"² compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1929.

PREVIOUS INVESTIGATIONS

Investigations of fluid flow in pipes have been concerned with the development of general formulas, and, more recently, with studies into the effect of artificial roughness on the friction factor.

General Considerations of Fluid Flow.—In the evolution of formulas dealing with the flow of fluid in pipes, the findings of Hagen, Poiseuille, Darcy, Reynolds, and Blasius are especially significant. In 1839, Hagen³ observed that there were two different types of fluid flow through pipes, now commonly designated as stream-line and turbulent flow. Thirty years later he stated that the point at which transfer from one form of flow to the other occurred depended on the radius, the velocity, and the temperature of the water.

In 1845, Poiseuille⁴ experimented with the flow of water through capillary tubes. He deduced the following law for stream-line flow, known as Poiseuille's law, which has since been derived theoretically:

$$F_l = \frac{64 \mu L V}{2 g D^2} \dots\dots\dots (1)$$

in which, F_l = loss of force, or pressure; μ = absolute viscosity; L = length of experimental pipe, in feet; V = average velocity, in feet per second; g = acceleration due to gravity; and D = mean diameter of pipe, in feet. Equation (1) may be expressed in terms of the Reynolds number and the Darcy friction factor, as follows:

$$f = \frac{64}{R} \dots\dots\dots (2)$$

in which, f = Darcy's friction factor; and R = Reynolds number.

In 1857, Darcy⁵ conducted a series of experiments on the flow of water in cast-iron pipes with diameters ranging from 0.5 in. to 20 in., obtaining the formula,

$$H_f = \frac{V^2 L}{c^2 D} \dots\dots\dots (3)$$

in which, H_f = loss of head due to friction; and c^2 = a coefficient which Darcy considered to be constant for each type of surface. The Darcy formula is now usually written in the form,

$$H_f = \frac{f L V^2}{2 g D} \dots\dots\dots (4)$$

² A. S. A.—Z10b—1929.

³ "Über die Bewegung des Wassers in engen zylindrischen Röhren," von G. Hagen, *Poggendorff's Annalen*, Bd. 46, S. 423, 1839.

⁴ "Recherches expérimentales sur le mouvement des liquides dans tubes de très petits diamètres," par Poiseuille, *Comptes Rendus*, Vol. 11, 1840, pp. 961, 1041; Vol. 12, 1841, p. 112; also, *Mémoires des Savants Etrangers*, Vol. 9, 1846.

⁵ "Recherches expérimentales relatives au mouvement de l'eau dans les tuyaux," par H. Darcy, *Mémoires des Savants Etrangers*, Vol. 15, 1858, p. 141.

in which, $f = \frac{2g}{c^2}$. The friction factor, f , will be used throughout this paper as the measure of frictional resistance.

By a study of Stokes' equations of motion, Reynolds⁶ conceived the idea that a criterion to determine whether flow would be stream-line or turbulent might be expressed in the form,

$$R = \frac{DV\rho}{\mu} \dots\dots\dots (5)$$

in which, ρ = density, in pounds per cubic foot. In 1883 he confirmed this theory experimentally by observing dye introduced into flowing water in glass tubes. The value of Reynolds number, $\frac{DV\rho}{\mu}$, at which change from one

flow form to the other takes place is modified more or less by conditions at the inlet of the pipe, by roughness, the amount of curvature, etc. A reasonable mean value to accept as the transition point between stream-line and turbulent flow is $R = 2100$.

Working with the experimental data of Saph and Schoder⁷ on flow of water in smooth brass pipe, Blasius⁸ obtained the empirical formula,

$$f = 0.316 R^{-0.25} \dots\dots\dots (6)$$

Reynolds numbers for these experiments were less than 100 000. Later experiments on smooth pipe show that the Blasius formula does not fit the data for Reynolds numbers greater than 100 000. Nikuradse⁹ obtained the relation,

$$f = 0.0032 + 0.221 R^{-0.237} \dots\dots\dots (7)$$

from his experiments on smooth pipe, with R as high as 3 200 000. Drew, Koo, and McAdams¹⁰ obtained the formula,

$$f = 0.0056 + 0.500 R^{-0.32} \dots\dots\dots (8)$$

from a study of all available experimental data on smooth pipes.

Experiments with Artificially Roughened Pipes.—During the fifteen years since about 1920, Schiller, Hopf, Fromm, and Nikuradse have experimented with artificially roughened pipes to determine whether roughness follows the law of similitude. In order to have similitude, corresponding surface measurements or dimensions of roughness elements in pipes must be in the same ratio as the radii, such that, if k is the measure of absolute roughness

⁶ "An Experimental Investigation of the Circumstances Which Determine Whether the Motion of Water Shall Be Direct or Sinuous, and of the Law of Resistance in Parallel Channels," by Osborne-Reynolds, *Philosophical Transactions*, Royal Soc., Lond, 1883; or *Scientific Papers*, Vol. II, p. 51.

⁷ *Transactions*, Am. Soc. C. E., Vol. LI (1903), p. 253.

⁸ "Das Ähnlichkeitsgesetz bei Reilungsvorgängen," von H. Blasius, *Physicalische Zeitschrift*, Vol. 12, p. 1175, 1911; or, *Forschungsarbeiten Ingenieur*, Heft 131.

⁹ "Gesetzmässigkeiten der turbulenten Strömung in glatten Röhren," von J. Nikuradse, Verein Deutscher Ingenieur, *Forschungsheft*, No. 356, 1932.

¹⁰ "The Friction Factor for Clean Round Pipes," by Drew, Koo, and McAdams, *Journal*, Am. Inst. of Chemical Engrs., Vol. 28, p. 56, 1932.

and r is the radius of the pipe, the ratio, $\frac{k}{r}$, is a constant. Factor k , expressed in inches, is a complete measure of the size, shape, and distribution of the individual roughness elements. As there is at present no method for measuring or defining roughness, the symbol, k , is hypothetical.

Schiller¹¹ experimented with artificially roughened pipe of three sizes: 8 mm, 16 mm, and 21 mm, in inside diameter (1 in. = 25.3998 mm). The roughness consisted of spiral threads cut into the inner surfaces. Two arrangements of threads were utilized. In one, the pitch was 0.8 mm, and the depth, 0.6 mm; and in the other, the pitch was 0.4 mm, and the depth, 0.3 mm. The results of Schiller's tests are shown in Fig. 1. Curve 3 indi-

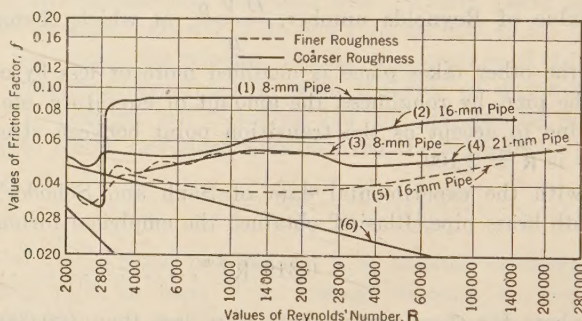


FIG. 1.—RESULTS OF EXPERIMENTS BY SCHILLER.

cates results for the 8-mm pipe with the finer roughness, and Curve 2, the 16-mm pipe with the coarser roughness. If the experiments on the two pipes conformed to the principles of similitude, Curves 3 and 2 should coincide since the roughness elements were proportional to the radii. The discrepancy may be attributed to the short calming lengths which appear to have been inadequate, and also to the fact that the roughness in the calming section of each tube differed from that in the test portion.

In investigating roughness in rectangular closed conduits, Ludwig Hopf¹² came to the conclusion that the friction factor depended upon three quantities: The cross-section of the conduit; Reynolds number; and a number which characterizes the roughness. He stated that the number for specifying the roughness appeared to require several dimensionless quantities to specify completely its size, shape, and distribution. Hopf also decided that there were two kinds of roughness, which he called "wall-waviness" and "wall-roughness." An example of the first is wood pipe or asphalted iron pipe, and an example of the second type is provided by the bare surface of cast-iron pipe.

Fromm's¹³ investigations of relative roughness were carried on shortly after those of Hopf and with the same experimental equipment. In each of

¹¹ "Über den Strömungswiderstand von Röhren verschiedenen Querschnitts und Rauheitsgrades," von L. Schiller, *Zeitschrift angewandte Mathematik und Mechanik*, Vol. 3, pp. 2-13, 1923.

¹² "Die Messung der hydraulischen Rauigkeit," von Ludwig Hopf, *Zeitschrift angewandte Mathematik und Mechanik*, Vol. 3, pp. 329-339, 1923.

¹³ "Strömungswiderstand in rauen Röhren," von K. Fromm, *Zeitschrift angewandte Mathematik und Mechanik*, Vol. 3, pp. 339-358, 1923.

the roughnesses tested, several values of the hydraulic radius were used. For those cases in which f becomes constant for larger values of R , the effect of the change of hydraulic radius may be expressed by the formula,

$$f = (\text{constant}) d^{-0.314} \dots \dots \dots (9)$$

in which, d is the depth of section, and the constant is dependent upon the absolute roughness, k' , of the surface.

Whereas Hopf, Fromm, and Schiller investigated relative roughness over a comparatively limited range of Reynolds numbers, Nikuradse¹⁴ obtained values of f within the range of turbulent flow up to a Reynolds number of 1 000 000. He used roughnesses varying from fairly smooth surfaces to projections equal to one-fifteenth of the radius. These roughnesses were produced by glueing sand to the inside of three sizes of pipes (diameters, 2.5 cm, 5 cm, and 10 cm). The diameter of the sand grain, k' , was taken as the measure of absolute roughness, k , the relative roughness being $\frac{k'}{r}$. Sand was selected of such

diameter that $\frac{k'}{r}$ was the same for the three different sizes of pipes. Six different values of $\frac{k'}{r}$ were used. The results of the experiments are shown in Fig. 2. The tests show that the principle of similitude applies accurately

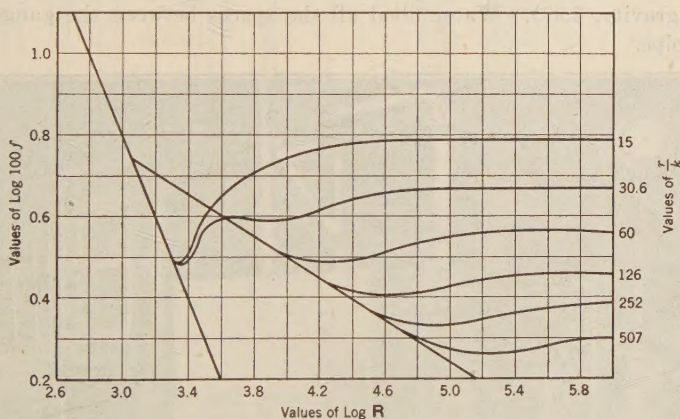


FIG. 2.—RESULTS OF EXPERIMENTS BY NIKURADSE.

to pipes roughened in this manner. For the range of Reynolds numbers in which f is constant, Nikuradse gives the formula,

$$f = \frac{1}{\left(1.74 + 2 \log_{10} \frac{r}{k'}\right)^2} \dots \dots \dots (10)$$

¹⁴ "Strömungsgesetze in rauen Röhren," von J. Nikuradse, Verein Deutscher Ingenieur, *Forschungsheft*, No. 361, 1933.

APPARATUS

The experiments undertaken in this investigation were conducted at the University of Michigan, Ann Arbor, Mich. The apparatus is illustrated in Figs. 3 and 4. The quantity of water used in the experiments varied from

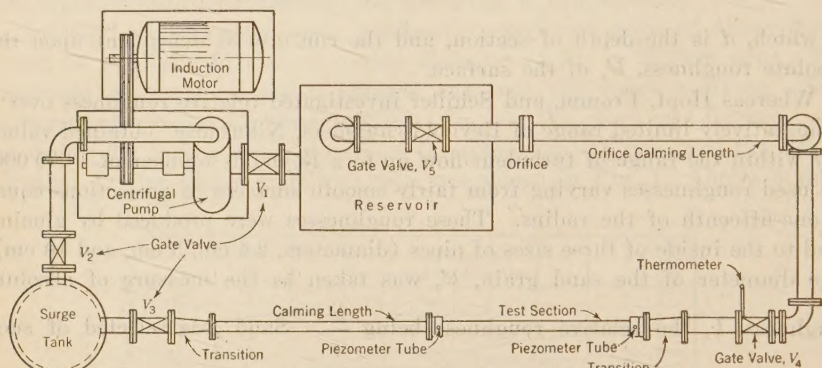


FIG. 3.—EXPERIMENTAL EQUIPMENT.

10 to 400 gal per min. Two orifices were used for measuring, one for small, the other for large, flows. Fig. 5 shows the orifice layout. Two differential manometers were used to measure the pressure drop across the orifice. One of the gauges contained mercury and the other acetylene tetra-bromide (specific gravity, 2.97). Water filled all the spaces between the gauge liquid and the pipe.

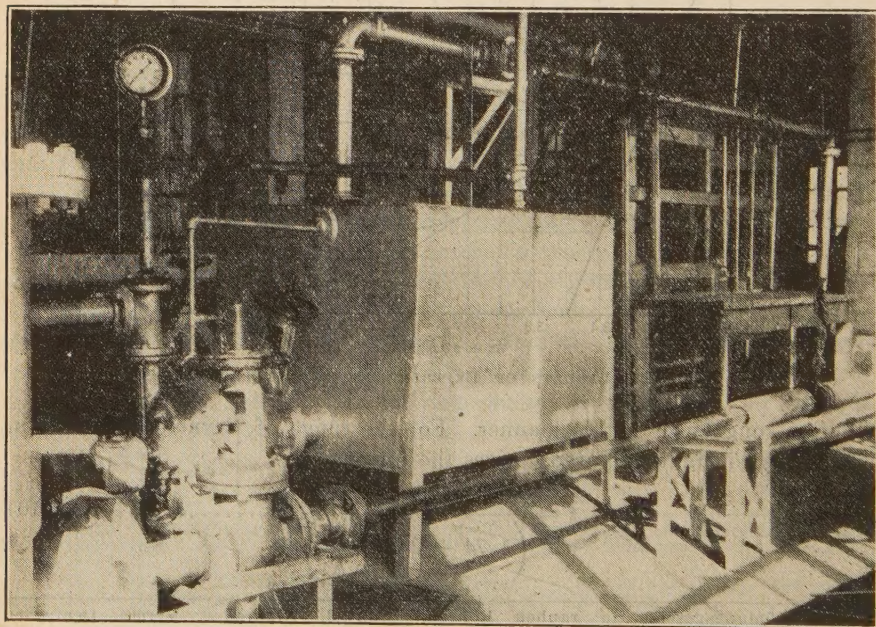


FIG. 4.—VIEW OF EXPERIMENTAL EQUIPMENT.

As the accuracy of piezometer rings is questionable in rough pipes, piezometer tubes of special design (see Fig. 6) were used to measure the pressure drop due to friction. These tubes were closed at the end, with a transverse

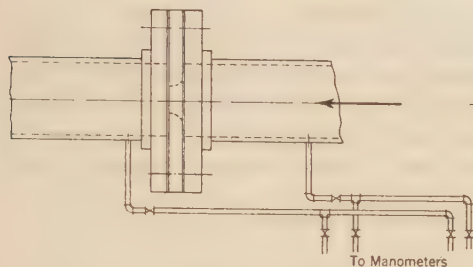


FIG. 5.—ORIFICE LAYOUT.

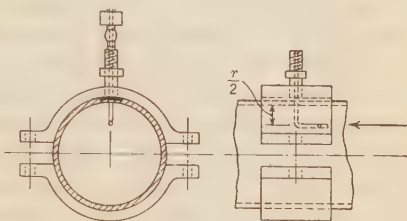


FIG. 6.—PIEZOMETER TUBE.

opening $\frac{1}{4}$ in. from the end of the tube. A diagram of the test-section manometers is shown in Fig. 7. Details of connections of copper tubing to manometer glass are illustrated in Fig. 8. Manometer *A*, Fig. 7, contained mercury; Manometer *B* contained acetylene tetra-bromide; and

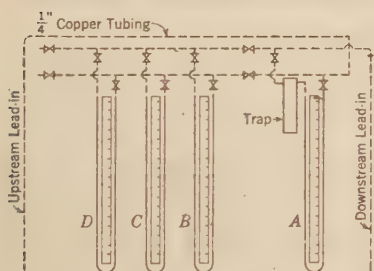


FIG. 7.—TEST SECTION, MANOMETER SYSTEM.

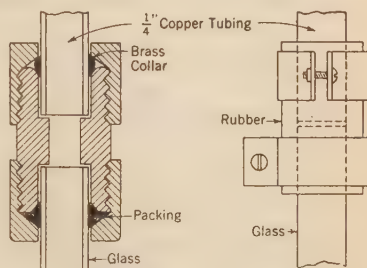


FIG. 8.—MANOMETER CONNECTIONS.

Manometers *C* and *D* contained solutions of acetylene tetra-bromide and xylene, with specific gravities, respectively, of 1.3 and 1.05. All of them were differential manometers with water acting as one liquid.

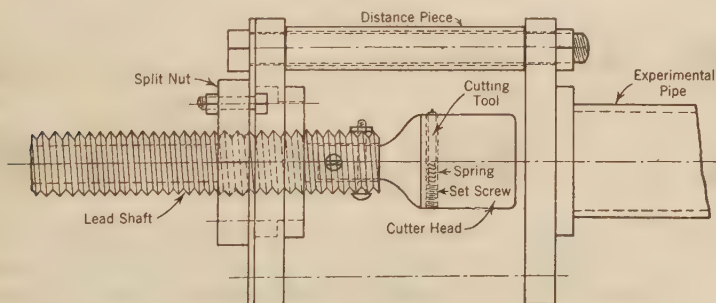


FIG. 9.—ROUGHENING APPARATUS.

A steam coil in the reservoir was used to heat the water to the desired temperature. A thermometer, reading to 110°C , was inserted in the valve at the down-stream end of the test section to obtain the temperature of water in the pipe (see Fig. 3). To insure isothermal flow through the test section, the pipe was insulated with felt covering.

In order to obtain a definitely known roughness on the inside surfaces of a 2-in. brass pipe, 10 ft. long, which would permit the size, shape, and distribution of the roughness to be varied, a device was designed that would cut grooves with various known dimensions and with different spacings. (See Fig. 9.) The pitch of the thread on the shaft was 11.5 to the inch. As the cutting tool could contain from one to four cutting elements within

$\frac{1}{11.5}$ in., the distribution of the grooves could be varied.

PRELIMINARY EXPERIMENTS

The preliminary experimental work included orifice calibrations, piezo-meter tests, and smooth pipe tests. The orifices were calibrated over the range of flows, for temperature of 72° and 158°F . Partial calibrations were made at temperatures of 55°F and 122°F . The experimental data for

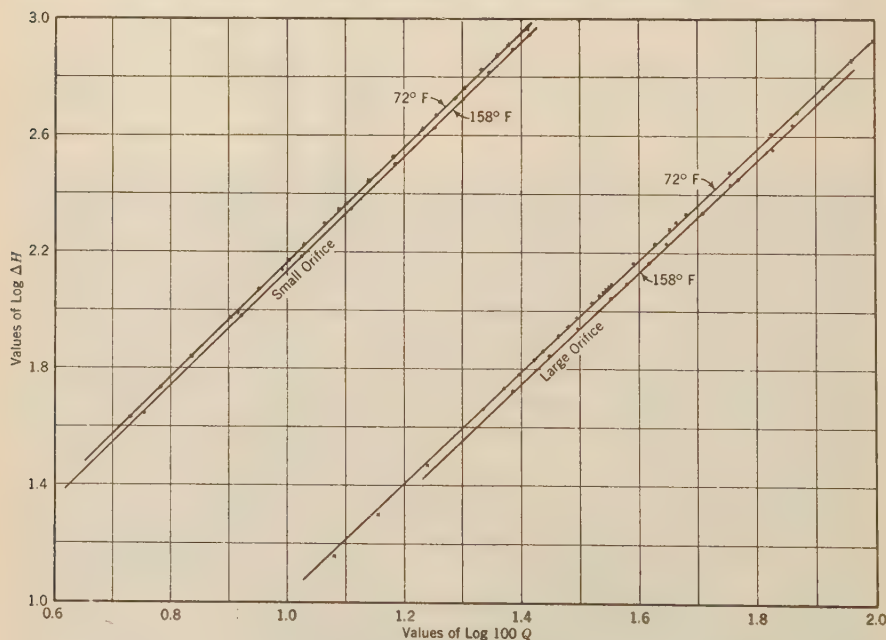


FIG. 10.—ORIFICE CALIBRATION.

temperatures of 72° and 158° are shown on the calibration curves in Fig. 10 (in which, ΔH represents the pressure drop across the orifice, in centimeters of water). Curves for 55° , 95° , and 122° were obtained from the curves for 72° and 158° , by assuming the change of discharge for a given pressure drop

across the orifice to be directly proportional to the change in viscosity of the water. A change of temperature from 55° to 158° increased the discharge by as much as 6.5% for a given pressure drop.

In order to be sure that the two piezometer tubes would give the correct difference in pressure in the test section, they were inserted into the same cross-section of a 2-in. pipe, and both were connected to a sensitive manometer. If the piezometer tubes had exactly the same characteristics and if the distribution of velocities in the cross-section were symmetrical, the manometer reading would be zero for any flow. These test readings were taken for several velocities, and they all showed the pressure drop across the

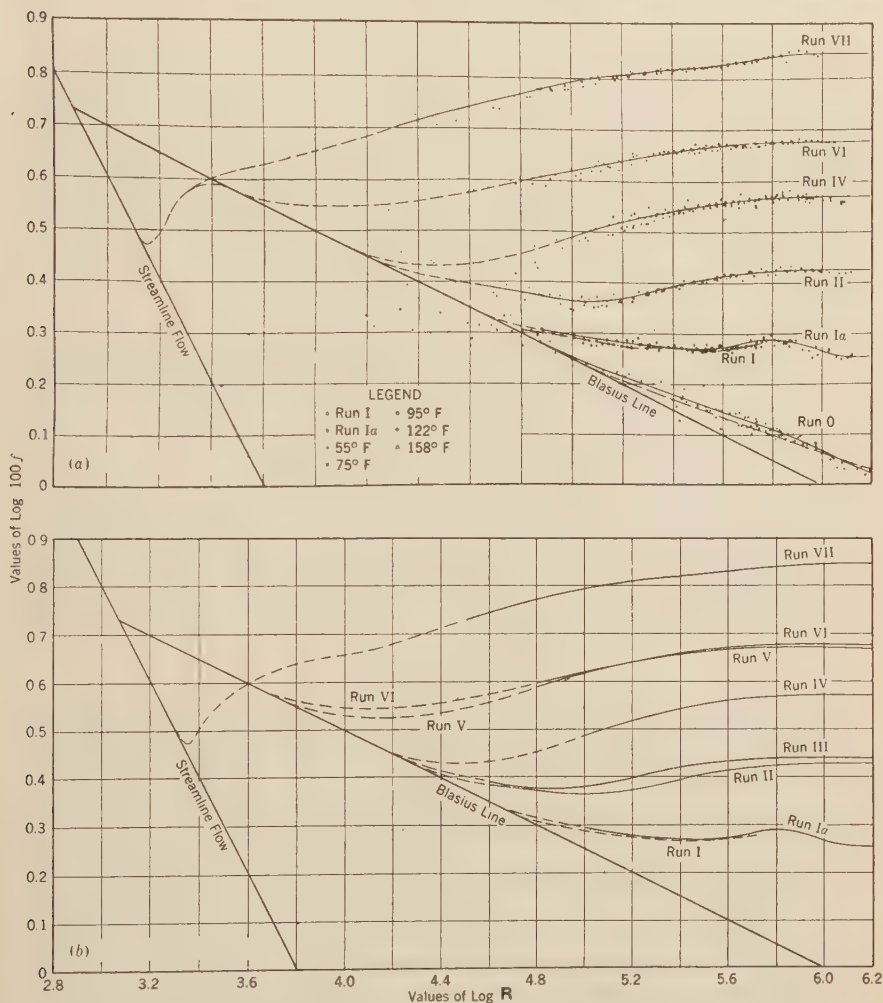


FIG. 11.—EXPERIMENTAL DATA: (a) FOR RUNS 0, I, Ia, II, IV, VI, AND VII; (b) CURVES OF f PLOTTED AGAINST R OBTAINED FROM TESTS ON ROUGHENED PIPES.

manometer to be small compared with the pressure drop at the same velocity that would occur between the two ends of the test section. Accordingly, it was not necessary to add a correction to the pressure-drop measurements.

Tests were made on smooth pipes in order to obtain a basis for comparison with other investigators, and also to enable operators to become familiar with the equipment. Two sets of experiments were made with drawn brass pipe to obtain curves showing the relation of f to the Reynolds number for different temperatures (see Run 0, Fig. 11(a)). These runs were conducted in the same manner as those on rough pipe.

EXPERIMENTS FOR OBTAINING DATA

The experiments on rough pipes consisted of roughening the pipes, determining the diameter, controlling the temperature, and taking readings. The test section was first roughened by running the cutter head (Fig. 9) through the pipe with the desired tool inserted in it. In order to obtain a sample of the roughness, the cutter head was run through a short piece of 2-in. brass pipe similar to that of the test section. After the sample was obtained, the 10-ft calming length was roughened in the same manner as the test section. Even with the greatest care, it was not possible to keep brass particles from collecting on the tool, so that the roughness became gradually less sharp as the cutter head advanced.

As the friction factor for a pipe varies as the fifth power of the diameter, it was important to determine the diameter accurately. This was done indirectly by determining the weight of water necessary to fill the 10-ft test section. With the weight of water, length of pipe, and density of water known, the mean diameter could be determined accurately.

The series of experiments performed on each test section was divided into eight runs, of which four were conducted with each orifice. Each run was made at a practically constant temperature. Runs were made with each orifice at temperatures of 55°, 95°, 122°, and 158° F. Each run consisted of readings of the orifice manometer, test-section manometer, and temperature for twelve to twenty different velocities.

COMPUTATIONS

In computing the friction factor, f , and the Reynolds number, R , the orifice manometer readings were converted into centimeters of water, and the test-section manometer readings into feet of water. From the proper orifice calibration curve the flow of water was read in cubic feet per second. The mean velocity of water in the test section was obtained by dividing the discharge by the mean area, in square feet. With this velocity (see first column, Table 2, Appendix II) the velocity head, $\frac{V^2}{2g}$, was obtained. Since $\frac{L}{D}$ is a constant for each roughness tested, when H_f (Appendix II) and V have been determined, the corresponding value of f can be computed from Equation (4) for each set of observations. The fraction, $\frac{\rho}{\mu}$, contained in the

Reynolds number is a function of the temperature, and, therefore, values of T can be traced through corresponding values in the third column of Table 2 (Appendix II). A table of values of $\frac{\rho}{\mu}$, in seconds per foot², was made up¹⁴ for each 0.1° C from 10 to 75 degrees. With V in feet per second, D , in feet, and $\frac{\rho}{\mu}$, in seconds per foot², the Reynolds number is seen to be dimensionless, and in consistent units. Finally, values of R and f , can be computed from the values listed in the fourth and fifth columns of Table 2 (see Appendix II).

DISCUSSION OF RESULTS

Seven artificial roughnesses were investigated, three of which were cut into one set (the test section and the calming length, Fig. 3) of brass pipes and the remaining four into another set. A roughness typified by very fine grooves was first used in each set of pipes and this was followed by roughnesses with successively coarser grooves. In each case an effort was made to choose roughness designs that would conform as closely as practicable to the design that was to be used next. In spite of every precaution, however, each roughness made the one that followed more irregular than it would have been if cut directly into smooth pipe.

Relation of Friction Factor to Reynolds Number.—The experimental values of $\log (100 f)$ and $\log R$ for Runs 0, I, Ia, II, IV, VI, and VII, are plotted in Fig. 11(a). As Run III had approximately the same friction factors as Run II, and Run V approximately the same as Run VI, their results are not given. The curves obtained from plotting $\log (100 f)$ against $\log R$ for all the runs on rough pipes are shown in Fig. 11(b). Run 0, Fig. 11(a), was made on smooth pipe. The dotted line through the smooth-pipe data is a graph of Equation (7). This line is seen to be in good agreement with the data. The left-hand portions of the curves of Figs. 11(a) and 11(b) are broken to indicate that these sections were not obtained experimentally, but constructed according to Nikuradse's curves (Fig. 2).

The runs were made at different temperatures with the twofold purpose of increasing the range of Reynolds numbers and of obtaining information concerning the value of f , for rough pipes, as a function of the Reynolds number.

As the ratio, $\frac{\mu (55^\circ)}{\mu (158^\circ)} = 2.98$, and as the density decreases very slightly with increases in temperature, changing the temperature of the water from 55° to 158° F increases the value of $\frac{\rho}{\mu}$ almost three times. With this temperature

change, therefore, it was possible to obtain a variation in Reynolds numbers almost three times greater than would be possible, for the same velocities, with water at one temperature. From the standpoint of results obtained, experimenting with water at these temperatures was practically equivalent to running tests

¹⁴ The values of ρ and μ were taken from "Elements of Chemical Engineering," by Badger and McCabe.

with other liquids which had corresponding values of $\frac{\rho}{\mu}$. To obtain the values indicated by any of the points on the curves shown in Fig. 11(a) would require velocities about three times as great if tests were run at a temperature of 55° F as would be required at a temperature of 158° F. In spite of the change in impact brought about by this velocity change, the friction factor remains the same. As comparatively few experiments have been conducted on artificially roughened pipes with viscosities other than those of water at temperatures between 32° and 75° F, these tests over a greater viscosity range furnish additional proof that, for a given rough pipe, f is a function of the Reynolds number.

Photo-Micrographic Study of the Roughnesses.—As a means of determining as closely as possible the shape of grooves in each test section, photo-micrographs, magnified to four times the actual size, of the pipe sample and cutting tool were taken for each roughness. In Figs. 13 and 14, photo-micrographs are magnified to 2.94 times their actual size. In some cases a groove intermediate between that of the cutting tool and the sample seemed to represent best the form of the roughness. In other cases (depending upon the number of previous cuts in the pipe, the wear on the cutting tool, and the

TABLE 1.—COMPARISON OF VALUES OF k' WITH ROUGHNESS DIMENSIONS

Roughness (see Fig. 12)							Roughness (see Fig. 12)						
(1)	(2) $f (= \text{a constant})$	(3) $\log \frac{r}{k'}$	(4) Radius, r , in inches	(5) Radius of sand grains, k' , in inches	(6) Average depth of grooves, in inches	(7) Number of grooves per inch	(1)	(2) $f (= \text{a constant})$	(3) $\log \frac{r}{k'}$	(4) Radius, r , in inches	(5) Radius of sand grains, k' , in inches	(6) Average depth of grooves, in inches	(7) Number of grooves per inch
I.....	0.0188	2.775	1.036	0.0017	0.005	46	IV.....	0.0370	1.727	1.043	0.0196	0.012	23
Ia.....	0.0188	2.775	1.036	0.0017	0.005	46	V.....	0.0465	1.45	1.043	0.0370	0.012	23
II.....	0.0266	2.195	1.034	0.0066	0.005	23	VI.....	0.0473	1.43	1.047	0.0390	0.016	11.5
III.....	0.0272	2.155	1.035	0.0072	0.005	23	VII.....	0.0700	1.02	1.052	0.1005	0.022	11.5

clearness of the photo-micrographs), the roughness appeared to be better represented either by the sample or by the tool. The number used herein to designate a type of roughness, corresponds to that used in Figs. 11(a) and

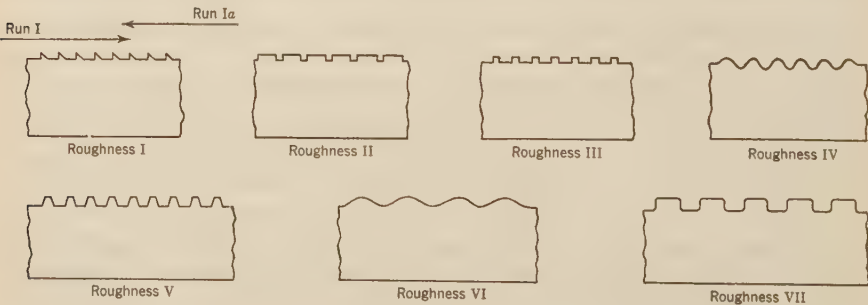


FIG. 12.—LONGITUDINAL SECTIONS OF ROUGHENED PIPE.

11(b) to indicate the run in which this roughness was used. Essential dimensions are classified in Table 1 and the roughness is shown diagrammatically for each case, in Fig. 12.

Roughness I.—The photo-micrographs of the sample and the tool for Roughness I are shown in Figs. 13(a) and 13(b), respectively. This was the

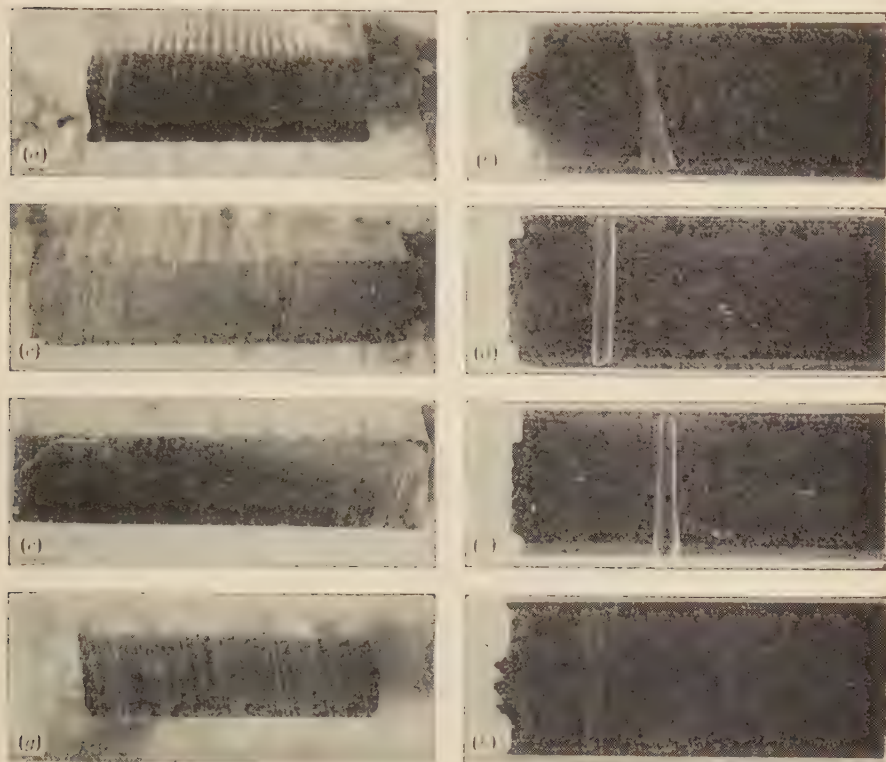


FIG. 13.—PHOTO-MICROGRAPHS OF LONGITUDINAL SECTIONS OF PIPE AND CUTTING TOOLS: *a*, *c*, *e*, AND *g*, RESPECTIVELY, DENOTE ROUGHNESSES I, II, III AND IV, WITH *b*, *d*, *f*, AND *g*, THE CORRESPONDING CUTTING DIES.

first cut through the first set of pipes, and, also, the finest roughness. Unfortunately, the sample photo-micrograph (Fig. 13(a)) does not show the profile of the roughness in such a way that measurements can be taken from it. The cutting tool (Fig. 13(b)), also, was badly damaged in going through the calming length, which had been slightly deformed as a result of some mishandling in transit. The arrows in Fig. 12 show the direction of flow. Run Ia was made with the same roughness, but with the direction of flow reversed.

Roughness II.—The photo-micrographs for Roughness II are shown in Fig. 13(c) and the cutting die in Fig. 13(d). It was the first cut into the second set of brass pipes, and, therefore, the shape and dimensions of

the roughness can be taken from the tool. The photo-micrograph of the sample does not aid much in obtaining the dimensions of the elements.

Roughness III.—The roughness indicated by Fig. 13(e) and the corresponding die, Fig. 13(f), was obtained by cutting over Roughness II with a tool designed to leave the roughness shown in Fig. 12. This was the only case of a roughness being cut to the same depth as that which preceded it. The cutting was probably 75% efficient (that is, 75% of pipe surface had the roughness shown by the tool, and the remainder was characterized by Roughness II). Roughness III as shown in Fig. 11(b) had a higher friction factor than Roughness II. This indicates that a projection causes greater turbulence than a groove.

Roughness IV.—Fig. 13(g) and the corresponding die, Fig. 13(h), show that this roughness varied in height from 0.018 in. at the end where the cutting began, to 0.006 in. at the other end of the pipe. This was the second roughness in the first set of pipes. The shape in Fig. 12 was a mean between tool and sample.

Roughness V.—The roughness, indicated by Fig. 14(a) and Fig. 14(b), had about the same average height as Roughness IV, and the same distribu-

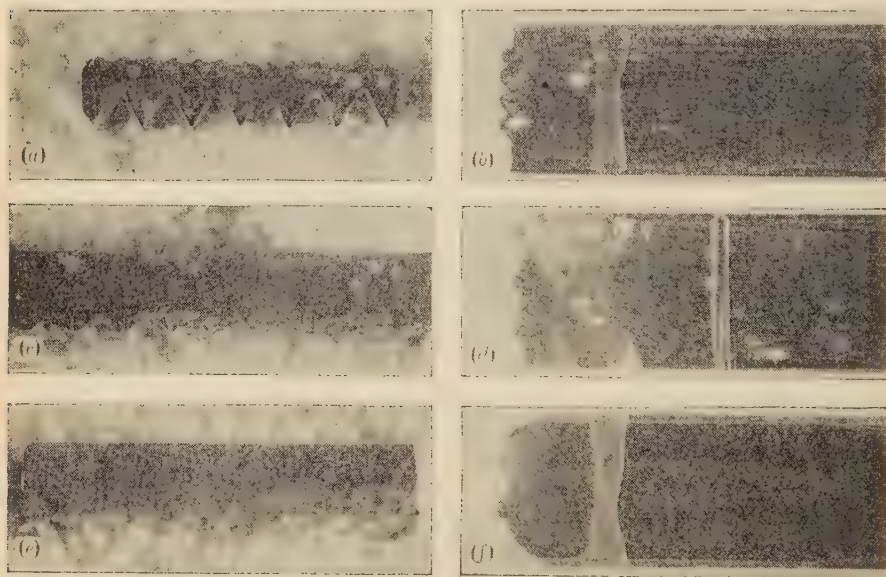


FIG. 14.—PHOTO-MICROGRAPHS OF LONGITUDINAL SECTIONS OF PIPE AND CUTTING TOOLS: *a*, *c*, AND *e*, RESPECTIVELY, DENOTE ROUGHNESSES V, VI AND VII, WITH *b*, *d*, AND *f*, THE CORRESPONDING CUTTING DIES.

tion, but was much more rough and abrupt in shape. This was the third roughness in the second set of pipes. The effect of shape is strikingly shown by the difference in friction factors of Roughnesses IV and V. In Fig. 14(a), the threads on the lower side are the standard for 2-in. pipes.

Roughness VI.—The third and last roughness of the first set of pipes (Roughness VI) is shown in Fig. 14(c) and Fig. 14(d). The shape was wavy in character, without sharp projections, as shown in Fig. 12. This roughness had a slightly greater friction factor than Roughness V, which had smaller elements and with twice as many peaks per inch. The decidedly abrupt shape of Roughness V seems to account for its friction factor being almost as large as that of Roughness VI.

Roughness VII.—Fig. 14(e) and Fig. 14(f) show Roughness VII, the fourth cut through the second set of pipes. As would be expected with its large grooves and sharp projections, this roughness gave the greatest friction factor of any shape investigated.

Relative Roughness.—While the description of the roughnesses and their comparisons have been made on the basis of actual size, it should be remembered that this can be done only because the radius of the pipe has changed little in comparison with the changes in roughness. This investigation neither proves nor disproves the similitude theory as applied to relative roughness. Nikuradse has made a satisfactory experimental demonstration of the validity of this theory.

The friction factor is a function of the Reynolds number and the relative roughness, or,

$$f = \phi \left(R, \frac{k}{r} \right) \dots \dots \dots (11)$$

The relative roughness factor, $\frac{k}{r}$, must have the same value for any two pipes

if they are to have the same values of f for all values of R . At present, there is no known method of determining k . It is undoubtedly some function of the size, shape, and distribution of individual roughness elements. The problem of determining numerical values of k that will define certain standard surfaces is an important one that will doubtless receive the attention of future investigators.

Comparison of Results with Those of Other Investigators.—A comparison with Nikuradse's experiments was made by investigating conditions in the region of large Reynolds numbers where the curve representing the mean of his results is approximately horizontal. Values of k' were computed from Nikuradse's formula (Equation (10)), the friction factor, f , being taken from the right side of the curves in Fig. 11(b). These values of k' are the diameters of sand grains required to give the same roughness effect as the corresponding grooves. Table 1 gives the value of k' obtained in this manner. In all cases, except Roughness I and Roughness Ia, the diameters of sand grains are larger than the depths of the grooves. The shape of the rough surfaces seems to have almost as much effect as the depth of the grooves. In Runs I and Ia the closeness of the roughness particles appears to account for the small friction factor. Run III has the same depth of groove as Run I, and the shape is not radically different; and yet the sand grains for Run III have about four times the diameter of those for Run I. Apparently, this increase in the value of f is caused entirely by the distribution of the roughness elements.

The ratio of the depth of groove to the radius of the pipe in Schiller's experiments was larger in all cases than those used in this investigation. The distributions used by Schiller were about 32 and 64 grooves per in. Because of the relatively larger grooves and the smaller range of Reynolds numbers covered by his experiments, it is difficult to establish relationships between the results of the two investigations.

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CONCLUSION

The experiments described herein indicate that the shape of the roughness elements is quite as important as the size. This is evident from an examination of Columns (5) and (6) of Table 1, as well as Fig. 11. Runs I, II, and III, while having the same depth of groove, have a wide variation in friction factor. As the shape of Runs I and III are somewhat similar, the difference in friction factor may be attributed to the difference in distribution of the roughness elements.

APPENDIX I

NOTATION

The following symbols, adopted for use in this paper, conform in all essential respects with "Symbols for Hydraulics" compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1929:

- c = coefficient; a constant for each type of roughened surface
(not to be confused with Chezy's coefficient, C).
- d = depth; depth of flow; depth of section.
- f = friction factor used in expressing loss of head in pipes; Darcy's friction factor; as a subscript, f denotes "friction."
- g = gravity constant; acceleration due to gravity.
- h = head; as a subscript, h denotes "hydraulic radius."
- k = a measure of absolute roughness; k' = diameter of sand grains, taken as a measure of absolute roughness.
- r = radius of pipe
- A = area; cross-section area of pipe
- C = Chezy's coefficient (not to be confused with c , as introduced in the paper).
- D = mean interior diameter of a pipe.
- F = force; total pressure; F_l = loss of pressure.
- H = total head; H_f = total loss of head due to friction.

L = length of experimental pipe sections.

R = hydraulic radius (not to be confused with $\mathbf{R}$ (see Equation (5))).

$\mathbf{R}$ = Reynolds number; $\mathbf{R}_h$ = Reynolds number based on the hydraulic radius

T = temperature.

V = velocity; average velocity in a section.

μ = viscosity, absolute.

γ = viscosity, kinetic $= \frac{\mu}{\rho}$.

ρ = density

ϕ = function of.

APPENDIX II

SUMMARY OF COMPUTATIONS

TABLE 2.—COMPUTATIONS TO DETERMINE FRICTION FACTOR, f , AND REYNOLDS NUMBER, $\mathbf{R}$

Mean velocity, V , in feet per second	Loss of head, H_f , due to friction, in feet	VALUES OF:			Mean velocity, V , in feet per second	Loss of head, H_f , due to friction, in feet	VALUES OF:		
		$\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{feet}^2}$	$\log \mathbf{R}$	$\log (100 f)$			$\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{feet}^2}$	$\log \mathbf{R}$	$\log (100 f)$
5.71	0.555	74 800	4.867	0.2975	18.19	3.99	126 440	5.598	0.148
1.85	0.059	74 800	4.377	0.3035	19.44	4.52	126 960	5.629	0.144
1.39	0.0514	74 800	4.253	0.492	20.35	4.94	127 740	5.651	0.143
3.14	0.159	74 000	4.602	0.274	21.2	5.34	127 740	5.669	0.145
3.84	0.26	73 200	4.685	0.314	22.15	5.74	128 260	5.69	0.135
4.243	0.333	78 800	4.760	0.334	22.8	6.06	128 580	5.704	0.136
5.45	0.488	80 060	4.876	0.282	24.36	6.85	129 300	5.734	0.132
5.895	0.570	80 480	4.9120	0.282	26.4	7.95	129 820	5.772	0.124
9.69	1.275	125 400	5.322	0.200	28.2	8.89	130 340	5.802	0.114
11.71	1.775	125 400	5.44	0.178	29.4	9.64	130 600	5.820	0.1125
13.13	2.175	125 400	5.454	0.166	30.45	10.2	130 600	5.836	0.108
14.59	2.635	125 400	5.499	0.159	32.38	11.4	130 860	5.862	0.104
15.75	3.05	125 660	5.532	0.156	34.1	12.57	130 860	5.886	0.101
16.3	3.21	125 660	5.548	0.1495	35.6	13.5	131 120	5.906	0.095
17.2	3.61	125 920	5.572	0.153	38.6	15.37	131 380	5.941	0.08

(a) RUN 0, PART a; $A = 0.0233$ SQUARE FEET; $D = 0.1722$ FEET; AND $\frac{L}{D} = 55.152$.

TABLE 2.—(Continued)

Mean velocity, V , in feet per second	Loss of head, H_f , due to friction, in feet	VALUES OF:			Mean velocity, V , in feet per second	Loss of head, H_f , due to friction, in feet	VALUES OF:		
		$\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{feet}^2}$	log R	log (100 f)			$\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{feet}^2}$	log R	log (100 f)
(b) RUN 0, PART b; $A = 0.0234$ SQUARE FEET; $D = 0.1720$ FEET; AND $\frac{L}{D} = 55.23$									
2.38	0.0945	99 550	4.610	0.288	7.19	0.5535	178 340	5.344	0.096
3.717	0.2224	100 700	4.808	0.273	8.72	0.878	178 340	5.428	0.128
4.96	0.3855	100 700	4.9345	0.2595	12.85	1.831	177 800	5.594	0.112
5.395	0.454	100 700	4.970	0.2595	17.08	3.186	177 800	5.718	0.1045
5.99	0.547	100 470	5.0144	0.249	19.86	4.273	178 340	5.784	0.102
6.64	0.6575	100 470	5.0593	0.240	23.56	5.909	177 800	5.858	0.094
7.67	0.850	100 700	5.1233	0.226	27.4	7.75	177 800	5.924	0.081
8.71	1.090	100 940	5.180	0.224	30.8	9.86	177 800	5.974	0.084
9.68	1.328	100 940	5.225	0.218	34.7	11.95	177 800	6.026	0.062
10.87	1.628	100 940	5.2755	0.2055	36.6	13.09	177 800	6.050	0.056
9.51	1.27	99 550	5.212	0.2135	39.6	14.49	177 800	6.090	0.031
13.20	2.31	99 320	5.353	0.205	8.68	0.911	229 000	5.534	0.150
15.93	3.24	99 550	5.436	0.172	13.5	2.02	229 810	5.728	0.111
18.42	4.16	99 780	5.50	0.155	17.34	3.255	229 540	5.836	0.100
21.52	5.69	99 780	5.3875	0.148	19.65	4.07	229 270	5.890	0.088
23.95	6.65	99 320	5.612	0.130	22.23	5.122	229 000	5.922	0.0795
27.36	8.45	99 090	5.668	0.120	24.0	5.85	229 270	5.976	0.073
30.20	10.30	99 550	5.7138	0.1195	27.44	7.295	229 540	6.0355	0.052
32.7	11.97	99 780	5.749	0.115	30.43	8.81	229 540	6.080	0.440
34.86	13.43	99 320	5.7754	0.1096	33.43	10.60	229 270	6.124	0.042
36.7	14.57	99 320	5.797	0.160	36.27	12.18	229 270	6.159	0.034
37.88	15.20	99 550	5.812	0.0918	38.62	13.60	229 810	6.194	0.027
8.90	0.975	170 240	5.416	0.156	6.84	0.625	232 700	5.437	0.192
12.78	1.918	169 700	5.572	0.136	8.28	0.907	233 300	5.522	0.198
16.36	3.028	169 700	5.679	0.120	13.71	2.163	232 000	5.738	0.127
18.8	3.97	169 700	5.740	0.1175	16.98	3.181	231 180	5.830	0.1105
21.2	5.00	169 700	5.792	0.112	20.47	4.506	231 730	5.912	0.098
23.33	5.885	169 970	5.834	0.100	23.6	5.790	232 700	5.976	0.082
26.58	7.40	170 240	5.892	0.088	27.6	7.41	232 700	6.044	0.055
29.8	9.045	169 700	5.940	0.074	30.92	9.10	232 700	6.093	0.0455
32.5	10.8	169 430	5.977	0.076	34.5	10.97	232 700	6.144	0.030
34.8	12.11	169 700	6.008	0.067	37.1	12.30	232 700	6.176	0.017
37.2	13.44	169 700	6.042	0.055	39.6	13.71	232 000	6.203	0.026
39.55	14.50	169 700	6.062	0.034					

(c) RUN I: $A = 0.2333$ SQUARE FEET; $D = 0.1727$ FEET; AND $\frac{L}{D} = 54.99$.

1.20	0.0265	75 600	4.195	0.332	9.88	1.567	91 600	5.193	0.2735
1.72	0.0625	76 000	4.354	0.393	10.43	1.747	91 600	5.218	0.272
2.35	0.1025	74 800	4.482	0.337	10.9	1.89	91 600	5.236	0.270
2.785	0.138	80 900	4.590	0.318	9.68	1.554	100 010	5.2214	0.2880
3.01	0.1595	83 210	4.636	0.314	10.80	1.90	99 780	5.2669	0.2788
3.43	0.203	85 100	4.703	0.3055	11.59	2.185	100 470	5.3032	0.2788
3.93	0.263	85 730	4.765	0.300	12.94	2.70	100 700	5.3522	0.2742
4.31	0.321	85 940	4.806	0.305	14.11	3.195	100 700	5.3838	0.2748
4.61	0.366	86 300	4.838	0.303	15.48	3.815	101 180	5.4322	0.2718
4.91	0.409	86 780	4.867	0.298	17.11	4.63	102 140	5.4800	0.2648
5.09	0.442	87 200	4.884	0.300	18.58	5.43	102 860	5.517	0.2648
5.51	0.5165	87 640	4.922	0.299	19.26	5.84	103 100	5.5353	0.2648
5.99	0.605	88 300	4.960	0.296	20.36	6.55	104 300	5.5697	0.2672
6.63	0.7305	89 180	5.009	0.290	21.56	7.44	105 020	5.5916	0.2768
7.20	0.858	89 620	5.046	0.288	23.07	8.45	105 740	5.6243	0.2695
7.54	0.934	90 060	5.068	0.284	25.18	10.2	106 220	5.6646	0.2742
7.96	1.02	90 500	5.095	0.2755	27.05	11.82	106 940	5.6981	0.2765
8.43	1.15	90 720	5.120	0.278	29.12	13.6	107 660	5.7332	0.2742
8.80	1.254	91 160	5.142	0.278	30.73	15.18	107 900	5.7574	0.2742
9.27	1.39	91 380	5.164	0.2765					

(d) RUN Ia: $A = 0.2333$ SQUARE FEET; $D = 0.1727$ FEET; AND $\frac{L}{D} = 54.99$

9.65	1.556	110 400	5.265	0.290	19.4	6.08	115 150	5.587	0.2765
11.28	2.073	110 900	5.334	0.2815	20.63	6.77	115 400	5.615	0.2695
13.21	2.818	111 900	5.407	0.276	21.76	7.55	115 650	5.638	0.272
14.78	3.46	112 150	5.457	0.268	22.6	8.14	116 900	5.658	0.272
16.15	4.16	112 900	5.498	0.2705	23.4	8.80	117 400	5.676	0.2735
17.31	4.79	113 400	5.531	0.2705	24.2	9.41	117 900	5.692	0.274
18.35	5.39	114 150	5.558	0.272	25.1	10.45	118 900	5.712	0.288

TABLE 2.—(Continued)

Mean velocity, V , in feet per second	Loss of head, H_f , due to friction, in feet	VALUES OF:			Mean velocity, V , in feet per second	Loss of head, H_f , due to friction, in feet	VALUES OF:		
		$\frac{\rho}{\mu}$, in seconds feet ²	log R	log (100 f)			$\frac{\rho}{\mu}$, in seconds feet ²	log R	log (100 f)
(d) RUN I _g : $A=0.2333$ SQUARE FEET; $D=0.1727$ FEET; AND $\frac{L}{D}=54.99$. (Continued)									
26.4	11.45	119 650	5.737	0.2835	9.1	1.336	119 150	5.272	0.276
27.78	12.69	120 150	5.760	0.284	8.28	1.130	121 400	5.240	0.2855
28.75	13.62	120 400	5.776	0.286	1.653	0.0515	74 200	4.326	0.343
29.65	14.48	121 650	5.794	0.298	7.60	0.922	144 380	5.278	0.272
30.55	15.45	122 400	5.810	0.288	7.555	0.908	152 700	5.299	0.270
31.5	16.40	123 150	5.826	0.286	10.72	1.801	160 500	5.473	0.263
32.4	17.15	124 900	5.844	0.281	10.67	1.785	180 500	5.522	0.265
32.93	17.91	130 340	5.870	0.287	10.57	1.760	196 080	5.552	0.266
32.93	17.88	132 940	5.877	0.285	10.53	1.750	201 680	5.564	0.266
14.43	3.25	130 600	5.513	0.261	9.34	1.437	85 310	5.138	0.285
17.66	4.89	142 820	5.638	0.263	11.07	2.023	86 780	5.219	0.287
21.28	7.245	147 500	5.734	0.272	13.17	2.835	87 200	5.297	0.282
24.3	9.54	154 000	5.810	0.276	15.7	3.964	94 260	5.408	0.276
26.27	11.29	157 900	5.855	0.281	17.82	5.144	100 010	5.489	0.277
28.21	13.20	161 800	5.896	0.287	18.97	5.815	104 300	5.534	0.278
33.04	16.92	167 540	5.982	0.258	20.3	6.645	128 000	5.652	0.275
30.55	14.26	174 020	5.962	0.252	22.0	7.812	137 880	5.718	0.276
33.12	17.17	185 160	6.026	0.2625	23.5	9.25	143 860	5.766	0.293
2.763	0.141	112 650	4.731	0.3335	33.64	17.84	151 660	5.946	0.265
3.87	0.2605	113 400	4.88	0.308	27.15	11.82	157 380	5.868	0.274
4.705	0.3741	112 650	4.961	0.296	29.8	14.16	162 320	5.922	0.2695
5.15	0.448	114 900	5.009	0.296	31.62	16.04	172 670	5.976	0.2735
5.58	0.506	114 650	5.042	0.280	33.6	17.91	182 660	6.027	0.2695
6.025	0.584	114 650	5.076	0.2755	33.77	17.40	194 960	6.058	0.252
7.25	0.866	115 150	5.098	0.286	25.62	10.30	199 440	5.946	0.262
7.506	0.931	115 400	5.175	0.286	33.78	17.29	205 040	6.08	0.2495
11.0	1.911	115 650	5.342	0.266	33.78	17.30	215 800	6.102	0.250
10.5	1.778	115 650	5.322	0.275	33.77	17.41	221 800	6.114	0.2525
9.96	1.611	117 150	5.305	0.278	33.78	17.50	224 700	6.120	0.255
(e) RUN II: $A=0.02331$ SQUARE FEET; $D=0.1723$ FEET; AND $\frac{L}{D}=55.14$.									
10.69	2.25	74 800	5.140	0.3615	12.49	3.32	128 000	5.440	0.396
9.80	1.90	75 000	5.103	0.3625	10.2	2.11	128 520	5.355	0.384
9.0	1.61	75 400	5.068	0.3655	8.34	1.442	128 000	5.265	0.3825
8.145	1.319	75 600	5.026	0.3645	10.42	2.286	128 000	5.362	0.3895
6.93	0.97	76 000	4.952	0.372	9.77	2.002	129 040	5.337	0.389
5.825	0.6845	76 600	4.886	0.372	8.98	1.662	129 040	5.300	0.3815
5.107	0.545	76 800	4.830	0.388	7.85	1.242	128 780	5.242	0.372
4.197	0.372	77 000	4.745	0.392	6.96	0.955	128 260	5.188	0.362
3.22	0.2192	77 400	4.634	0.392	5.97	0.684	127 480	5.119	0.3505
2.143	0.107	78 000	4.460	0.435	5.08	0.496	127 740	5.048	0.3505
1.582	0.0613	78 400	4.330	0.456	4.073	0.3185	128 000	4.954	0.3505
2.575	0.1421	79 010	4.545	0.398	2.96	0.174	128 000	4.815	0.364
3.615	0.2665	79 220	4.692	0.3765	2.283	0.1065	128 000	4.703	0.378
4.70	0.4504	78 800	4.806	0.376	3.482	0.232	128 000	4.886	9.348
5.505	0.615	78 400	4.872	0.374	4.565	0.3945	128 000	5.004	0.344
33.4	24.95	76 000	5.642	0.416	5.55	0.5905	128 000	5.088	0.350
31.6	22.1	76 200	5.618	0.411	10.6	2.44	167 540	5.486	0.4035
29.76	19.5	76 400	5.593	0.410	9.85	2.092	166 480	5.452	0.400
28.0	17.2	76 600	5.568	0.4075	9.04	1.73	166 220	5.414	0.392
25.95	14.61	76 800	5.536	0.404	8.025	1.34	166 480	5.363	0.386
24.18	12.54	76 800	5.505	0.3985	7.04	1.011	166 740	5.306	0.3765
22.23	10.63	77 000	5.470	0.400	6.09	0.7445	167 000	5.244	0.369
20.36	8.82	77 400	5.434	0.396	3.91	0.300	166 220	5.051	0.360
18.4	7.125	77 600	5.391	0.391	2.86	0.1665	165 180	4.911	0.375
16.52	5.72	77 800	5.346	0.388	2.122	0.100	164 920	4.781	0.414
14.47	4.33	77 800	5.288	0.383	3.462	0.228	164 920	4.994	0.346
12.58	3.276	78 000	5.228	0.3825	4.445	0.381	165 700	5.104	0.352
10.86	2.403	78 000	5.165	0.376	5.505	0.599	165 960	5.197	0.362
9.35	1.753	78 200	5.10	0.370	33.9	26.06	167 000	5.989	0.422
33.43	25.62	128 520	5.870	0.426	32.1	23.4	167 000	5.966	0.424
32.0	23.42	128 520	5.851	0.425	29.86	20.2	167 000	5.934	0.4215
29.7	19.7	128 000	5.816	0.416	27.36	17.02	167 000	5.896	0.424
27.2	16.54	128 520	5.781	0.416	25.2	14.37	167 000	5.860	0.422
24.15	12.94	128 000	5.726	0.414	22.75	11.79	167 000	5.816	0.4255
21.6	10.4	128 520	5.6805	0.415	19.9	9.07	167 000	5.758	0.428
19.36	8.30	128 520	5.632	0.413	18.11	7.43	167 000	5.717	0.422
17.1	6.39	128 000	5.577	0.408	15.75	5.535	167 000	5.656	0.4145
14.7	4.64	128 000	5.511	0.398	13.21	3.82	167 000	5.580	0.4065

TABLE 2.—(Continued)

Mean velocity, V , in feet per second	Loss of head, H_f , due to friction, in feet	VALUES OF:			Mean velocity, V , in feet per second	Loss of head, H_f , due to friction, in feet	VALUES OF:		
		$\frac{\rho}{\mu}$, in seconds feet ²	log R	log (100 f)			$\frac{\rho}{\mu}$, in seconds feet ²	log R	log (100 f)
(e) RUN II: $A = 0.02331$ SQUARE FEET; $D = 0.1723$ FEET; AND $\frac{L}{D} = 55.14$. (Continued)									
11.53 9.64 34.0 32.2 30.4 27.7 25.6 23.55 21.44 19.44 17.6 15.3 13.17 11.11	2.886 2.027 26.02 23.24 20.62 17.35 14.87 12.68 10.6 8.75 7.16 5.345 3.94 2.777	167 000 167 000 222 900 222 900 222 900 222 900 222 900 222 900 222 900 222 900 222 900 222 900 222 900 222 900	5.521 5.443 6.116 6.093 6.068 6.028 5.994 5.957 5.917 5.874 5.831 5.776 5.704 5.631	0.402 0.405 0.420 0.418 0.4165 0.420 0.422 0.427 0.429 0.432 0.431 0.424 0.424 0.418	10.6 9.84 9.00 7.975 6.94 6.02 5.11 4.005 2.82 1.902 3.55 4.51 5.595	2.53 2.157 1.778 1.372 1.010 0.757 0.532 0.318 0.160 0.0825 0.244 0.4045 0.6455	221 700 222 900 222 900 222 900 222 900 222 900 222 900 222 900 222 900 222 900 223 200 223 200 222 900	5.608 5.578 5.540 5.487 5.426 5.365 5.294 5.188 5.036 4.865 5.136 5.240 5.332	0.4185 0.414 0.408 0.402 0.3895 0.388 0.376 0.364 0.3705 0.4255 0.354 0.364 0.381
(f) RUN III: $A = 0.02336$ SQUARE FEET; $D = 0.1725$ FEET; AND $\frac{L}{D} = 55.09$.									
31.96 30.2 28.55 26.58 24.46 22.45 20.55 18.53 16.4 14.21 11.36 9.07 10.77 10.06 9.245 8.43 7.55 6.55 5.495 4.56 3.74 2.87 2.078 1.35 1.628 2.483 3.295 4.16 5.10 32.0 30.08 28.22 25.83 22.72 20.81 18.75 16.81 14.73 12.28 10.43 9.325 10.67 9.71 8.94 8.095 7.05 6.02 4.992 3.92 2.857 2.32 3.437 4.425	23.77 21.19 18.7 16.1 13.58 11.45 9.505 7.69 5.91 4.48 2.819 1.74 2.442 2.13 1.771 1.473 1.17 0.875 0.62 0.4345 0.2922 0.174 0.087 0.0467 0.0639 0.1251 0.216 0.356 0.5395 24.2 21.33 18.51 15.41 12.18 10.31 8.345 6.64 5.055 3.48 2.492 0.978 2.522 2.10 1.733 1.416 1.052 0.7355 0.505 0.302 0.1549 0.108 0.224 0.396	74 600 74 800 75 200 75 200 76 000 76 200 76 400 76 600 77 000 77 400 77 600 77 800 77 800 77 800 77 800 77 800 78 000 78 200 78 600 78 600 78 800 79 010 79 220 79 430 80 060 80 900 80 900 80 480 80 060 126 180 127 480 128 000 128 000 128 260 128 260 128 000 128 000 128 000 128 000 128 000 128 000 128 000 128 000 128 520 128 000 128 000 128 000 128 520 128 520 128 520 128 780	5.614 5.590 5.580 5.538 5.507 5.470 5.433 5.388 5.339 5.277 5.183 5.086 5.160 5.130 5.094 5.054 5.006 4.946 4.870 4.790 4.706 4.592 4.454 4.267 4.352 4.540 4.662 4.672 4.848 5.844 5.821 5.795 5.758 5.701 5.663 5.617 5.570 5.513 5.433 5.353 5.314 5.370 5.338 5.301 5.257 5.197 5.126 5.042 4.937 4.800 4.712 4.882 4.992	0.434 0.434 0.428 0.426 0.422 0.424 0.420 0.418 0.410 0.413 0.407 0.393 0.392 0.390 0.384 0.384 0.3795 0.378 0.381 0.387 0.388 0.392 0.372 0.476 0.450 0.375 0.366 0.380 0.3845 0.4415 0.439 0.4325 0.430 0.440 0.444 0.443 0.438 0.434 0.4315 0.426 0.4235 0.414 0.415 0.4035 0.402 0.393 0.376 0.374 0.361 0.346 0.370 0.346 0.373	5.58 6.02 32.02 30.35 28.4 26.46 24.02 22.0 20.0 18.09 16.0 13.5 11.36 9.8 9.2 10.74 10.07 9.19 8.243 7.225 6.195 5.20 4.126 3.083 1.964 1.61 4.72 5.725 32.4 30.73 28.82 27.12 24.83 23.0 20.86 18.58 16.54 14.0 11.7 10.89 10.07 9.30 8.29 7.20 6.17 5.20 4.163 3.303 2.571 1.893 3.67 4.69 5.745	0.631 0.736 24.11 21.38 18.69 16.16 13.40 11.37 9.40 7.61 5.89 4.192 2.957 2.186 1.928 2.655 2.281 1.874 1.48 1.149 0.796 0.571 0.359 0.1988 0.0890 0.0624 0.4463 0.672 24.22 21.61 19.29 17.03 14.37 12.38 10.29 8.14 6.48 4.577 3.218 2.727 2.323 1.978 1.549 1.141 0.846 0.585 0.3625 0.2325 0.1372 0.081 0.2656 0.469 0.720	128 000 128 000 166 220 166 480 166 480 167 000 166 480 166 480 166 740 166 740 166 740 167 000 167 000 166 740 166 480 169 700 169 160 168 350 168 350 168 350 167 270 167 270 167 270 167 270 167 000 167 540 167 540 167 000 165 700 222 900 221 400 221 400 221 400 222 900 222 900 222 900 222 900 222 900 222 900 224 280 223 420 221 400 221 400 221 400 221 400 222 000 222 900 222 900 222 900 222 900 223 420 222 900 222 900	5.091 5.124 5.964 5.940 5.912 5.882 5.839 5.801 5.760 5.716 5.663 5.590 5.515 5.450 5.423 5.498 5.468 5.426 5.378 5.321 5.252 5.176 5.076 4.95 4.754 4.668 5.133 5.214 6.096 6.070 6.042 6.018 5.980 5.947 5.905 5.854 5.804 5.732 5.654 5.626 5.589 5.550 5.501 5.439 5.373 5.298 5.203 5.104 4.995 4.862 5.150 5.257 5.344	0.374 0.376 0.4365 0.433 0.432 0.430 0.433 0.437 0.438 0.435 0.430 0.428 0.428 0.424 0.428 0.420 0.414 0.406 0.410 0.3845 0.392 0.3905 0.3875 0.431 0.449 0.369 0.380 0.431 0.428 0.432 0.432 0.434 0.436 0.440 0.440 0.442 0.435 0.438 0.4295 0.428 0.426 0.420 0.412 0.4145 0.403 0.388 0.3965 0.385 0.362 0.392 0.4065

TABLE 2.—(Continued)

Mean velocity, V , in feet per second	Loss of head, H_f , due to friction, in feet	VALUES OF:			Mean velocity, V , in feet per second	Loss of head, H_f , due to friction, in feet	VALUES OF:		
		$\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{foot}^2}$	$\log R$	$\log (100 f)$			$\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{foot}^2}$	$\log R$	$\log (100 f)$
8.87	2.275	94 030	5.162	0.530	15.68	7.51	170 240	5.666	0.5565
10.79	3.395	94 490	5.248	0.536	12.8	4.97	170 510	5.580	0.551
12.19	4.424	95 180	5.304	0.548	9.82	2.89	170 780	5.464	0.5475
14.19	6.025	95 870	5.372	0.546	5.77	0.969	170 780	5.234	0.534
16.8	8.51	96 560	5.450	0.550	30.2	28.45	170 780	5.952	0.566
18.8	10.81	97 710	5.504	0.554	30.22	28.6	170 780	5.953	0.566
20.75	13.13	98 400	5.550	0.556	29.4	27.1	171 320	5.942	0.5665
22.86	16.18	99 320	5.596	0.564	7.66	1.825	219 000	5.465	0.564
29.61	27.83	100 240	5.821	0.572	30.56	28.72	225 000	6.077	0.5585
26.44	22.28	102 620	5.674	0.574	30.66	28.95	229 000	6.087	0.558
27.9	24.36	105 500	5.709	0.565	1.861	0.0805	222 900	4.858	0.4365
30.4	29.28	112 900	5.775	0.571	2.702	0.188	228 100	5.030	0.481
30.35	28.68	128 000	5.829	0.564	3.639	0.352	229 270	5.162	0.4945
29.17	26.66	128 260	5.813	0.567	4.355	0.52	230 100	5.242	0.508
27.96	24.67	129 560	5.800	0.570	4.95	0.685	229 000	5.295	0.517
26.63	22.20	130 340	5.782	0.565	5.50	0.8825	228 550	5.340	0.536
25.0	19.40	128 520	5.747	0.563	6.46	1.22	228 100	5.409	0.5365
23.14	16.59	127 480	5.709	0.562	7.003	1.442	228 100	5.444	0.5395
21.4	14.19	125 400	5.669	0.562	7.70	1.756	227 650	5.483	0.542
30.4	29.01	120 400	5.803	0.566	8.48	2.158	227 650	5.525	0.5475
3.168	0.2515	129 560	4.854	0.469	9.008	2.445	228 100	5.553	0.5495
3.84	0.380	130 340	4.940	0.482	9.47	2.735	229 270	5.578	0.554
4.50	0.5295	129 560	5.005	0.487	10.1	3.11	229 270	5.605	0.555
5.005	0.670	127 740	5.046	0.498	10.87	3.618	229 270	5.637	0.556
5.36	0.774	127 480	5.076	0.5005	11.63	4.06	229 270	5.666	0.548
5.645	0.870	127 480	5.097	0.506	13.6	5.54	230 370	5.736	0.548
6.305	1.10	128 260	5.148	0.513	16.0	7.90	229 540	5.806	0.559
6.84	1.31	129 040	5.186	0.518	18.47	10.6	229 000	5.866	0.5635
7.54	1.61	129 040	5.228	0.523	20.9	13.77	228 550	5.920	0.569
8.18	1.92	129 040	5.264	0.528	23.2	16.80	228 550	5.965	0.5645
8.70	2.167	129 040	5.291	0.528	25.5	20.22	228 550	6.006	0.562
9.21	2.443	129 040	5.316	0.531	27.33	23.1	228 100	6.035	0.5605
9.71	2.745	128 780	5.337	0.535	30.2	27.82	228 100	6.078	0.5545
10.29	3.096	128 520	5.362	0.536	6.7	1.330	227 650	5.423	0.543
10.81	3.450	128 000	5.382	0.5395	10.28	3.207	228 550	5.611	0.552
3.147	0.2464	167 000	4.962	0.466	13.52	5.51	229 540	5.733	0.5505
4.005	0.4146	166 740	5.065	0.482	17.53	9.70	229 270	5.845	0.570
4.815	0.615	166 740	5.145	0.494	20.03	12.76	229 000	5.902	0.572
5.445	0.811	167 000	5.199	0.5075	21.8	15.17	229 270	5.939	0.574
5.71	0.8975	167 000	5.220	0.510	23.7	17.68	229 540	5.976	0.5685
6.04	1.011	168 080	5.246	0.514	27.0	22.82	229 000	6.031	0.566
6.485	1.71	167 270	5.276	0.516	30.4	28.32	229 000	6.082	0.557
7.365	1.541	166 740	5.328	0.5235	2.86	0.2143	227 380	5.053	0.4885
7.825	1.760	166 740	5.355	0.529	4.075	0.451	229 000	5.210	0.5035
8.445	2.063	167 000	5.389	0.532	5.00	0.699	230 100	5.302	0.5165
9.19	2.482	167 000	5.426	0.5385	5.435	0.855	229 540	5.336	0.5335
9.55	2.71	166 740	5.442	0.544	6.105	1.092	228 550	5.386	0.538
10.12	3.05	167 000	5.469	0.544	6.78	1.353	228 100	5.430	0.5405
10.72	3.46	167 000	5.494	0.598	7.56	1.714	228 100	5.478	0.548
14.98	6.79	167 270	5.640	0.551	8.70	2.296	229 000	5.540	0.552
16.85	8.72	167 270	5.690	0.558	9.46	2.760	229 820	5.577	0.559
18.6	10.7	167 270	5.734	0.561	10.23	3.228	229 540	5.612	0.560
20.5	13.28	167 270	5.776	0.5685	11.03	3.772	229 540	5.644	0.561
22.25	15.68	167 540	5.812	0.570	30.45	28.42	236 200	6.098	0.556
23.9	18.39	166 740	5.841	0.578	28.64	25.56	237 640	6.073	0.562
26.0	21.22	166 480	5.877	0.566	27.3	23.1	238 020	6.053	0.5615
27.95	24.38	166 480	5.908	0.564	25.82	20.8	237 640	6.028	0.564
29.6	27.05	166 740	5.934	0.5595	24.1	18.0	237 640	5.998	0.562
30.75	28.99	167 000	5.951	0.557	22.0	15.23	236 880	5.956	0.569
29.21	26.67	170 510	5.938	0.564	19.85	12.46	237 640	5.913	0.5765
28.36	25.41	170 780	5.925	0.570	17.83	10.08	237 640	5.867	0.572
27.17	23.2	170 780	5.906	0.568	15.27	7.225	237 640	5.799	0.562
25.15	20.42	171 050	5.872	0.580	11.68	4.245	237 640	5.683	0.563
23.5	17.39	170 510	5.843	0.568	7.99	2.00	236 880	5.517	0.5665
21.13	14.21	170 240	5.799	0.5665	5.61	0.921	236 880	5.356	0.552
19.03	11.31	170 240	5.750	0.565					

(g) RUN IV: $A = 0.02375$ SQUARE FEET; $D = 0.1739$ FEET; AND $\frac{L}{D} = 54.63$.

TABLE 2.—(Continued)

Mean velocity, V , in feet per second	Loss of head, H_f , due to friction, in feet	VALUES OF:			Mean velocity, V , in feet per second	Loss of head, H_f , due to friction, in feet	VALUES OF:		
		$\frac{P}{\mu}$, in seconds per foot ²	log R	log (100 f)			$\frac{P}{\mu}$, in seconds per foot ²	log R	log (100 f)
(h) RUN V: $A=0.02376$ SQUARE FEET; $D=0.1739$ FEET; AND $\frac{L}{D}=54.62$.									
10.07	3.761	75 600	5.122	0.640	28.37	31.75	167 000	5.916	0.667
11.62	5.05	75 600	5.184	0.644	27.10	28.75	167 000	5.896	0.663
12.52	5.91	75 600	5.218	0.6465	25.62	25.71	166 740	5.871	0.663
14.88	8.35	75 800	5.283	0.648	23.8	22.18	166 740	5.839	0.664
17.15	11.20	76 000	5.355	0.652	22.0	19.18	167 000	5.806	0.670
19.10	14.03	76 000	5.401	0.656	19.96	15.83	167 000	5.763	0.670
20.96	17.0	76 000	5.442	0.660	17.84	12.63	167 000	5.714	0.670
23.07	20.83	76 400	5.486	0.664	16.03	10.0	167 000	5.669	0.661
28.14	30.83	76 600	5.572	0.662	14.10	7.66	166 480	5.611	0.657
26.79	27.87	76 800	5.554	0.6605	10.20	4.005	166 220	5.470	0.657
25.20	24.78	77 200	5.530	0.662	11.10	4.708	166 220	5.507	0.654
23.96	22.25	77 400	5.508	0.660	12.21	5.75	166 480	5.548	0.6565
1.241	0.0372	78 000	4.226	0.454	1.241	0.0362	166 480	4.556	0.4425
1.509	0.0583	78 000	4.310	0.4695	2.182	0.1432	167 000	4.802	0.550
2.238	0.1419	79 010	4.488	0.524	3.097	0.3123	169 430	4.960	0.584
3.031	0.276	79 430	4.622	0.549	4.10	0.586	169 430	5.082	0.614
3.95	0.509	79 010	4.735	0.584	5.105	0.929	167 000	5.172	0.623
4.94	0.8315	78 600	4.830	0.604	4.60	0.745	166 220	5.124	0.6175
4.465	0.6805	78 400	4.786	0.604	3.565	0.4325	166 480	5.014	0.602
3.56	0.415	78 600	4.687	0.586	2.762	0.248	166 740	4.904	0.583
2.675	0.2162	78 800	4.564	0.552	1.691	0.0885	166 740	4.690	0.563
1.98	0.1186	79 220	4.436	0.553	5.675	1.172	167 000	5.217	0.632
1.443	0.0643	79 430	4.299	0.560	10.80	4.565	166 740	5.496	0.6635
5.54	1.062	79 430	4.883	0.6105	9.80	3.73	166 740	5.453	0.660
10.48	4.05	78 800	5.157	0.638	8.955	3.08	167 000	5.416	0.656
9.55	3.335	79 010	5.118	0.634	8.125	2.504	167 000	5.373	0.650
8.70	2.73	79 220	5.078	0.628	7.125	1.907	167 000	5.316	0.646
7.755	2.166	79 220	5.028	0.628	6.30	1.453	167 000	5.270	0.6355
6.88	1.69	79 220	4.978	0.624	28.3	31.64	222 900	6.040	0.668
6.08	1.295	79 430	4.924	0.616	27.22	29.0	222 900	6.024	0.6625
28.2	31.59	128 000	5.798	0.669	25.8	26.23	222 900	6.000	0.666
26.85	28.3	128 000	5.776	0.664	24.05	23.73	222 900	5.969	0.658
25.4	25.19	128 000	5.752	0.662	22.1	19.4	222 900	5.933	0.671
23.3	21.1	128 000	5.715	0.660	20.2	16.28	222 900	5.894	0.6715
21.48	18.09	128 000	5.680	0.664	18.23	13.2	222 900	5.850	0.670
19.52	14.91	128 000	5.638	0.664	16.08	10.27	222 900	5.795	0.670
17.6	11.97	128 000	5.594	0.658	13.85	7.445	222 900	5.730	0.660
15.72	9.46	128 000	5.544	0.654	10.28	4.23	222 900	5.601	0.6725
13.47	6.97	128 000	5.477	0.656	11.37	5.07	222 900	5.644	0.664
11.92	5.407	128 000	5.422	0.6515	12.2	5.845	222 900	5.675	0.6645
11.09	4.65	128 000	5.392	0.649	10.42	4.29	222 900	5.607	0.666
10.28	4.00	128 000	5.359	0.649	9.655	3.673	223 560	5.574	0.667
1.158	0.0306	130 080	4.418	0.431	9.04	3.182	223 920	5.546	0.661
2.38	0.171	131 120	4.735	0.5505	8.250	2.636	223 920	5.506	0.6595
3.10	0.3115	133 200	4.856	0.581	7.325	2.05	223 560	5.454	0.654
3.82	0.4905	132 680	4.944	0.598	6.535	1.612	223 200	5.405	0.6495
4.95	0.8625	130 080	5.048	0.618	5.92	1.327	222 900	5.362	0.650
4.44	0.683	129 560	5.000	0.610	5.405	1.10	222 900	5.322	0.648
3.60	0.4375	129 560	4.910	0.599	4.395	0.706	222 900	5.232	0.6325
2.70	0.2322	129 300	4.784	0.574	3.421	0.4195	222 900	5.123	0.626
2.024	0.1291	129 300	4.659	0.570	2.703	0.2475	222 900	5.021	0.600
5.50	1.065	129 040	5.092	0.6175	1.831	0.1101	222 900	4.852	0.588
10.81	4.49	127 740	5.380	0.655	1.211	0.0458	222 900	4.672	0.5655
9.92	3.76	127 740	5.342	0.652	1.372	0.0618	222 900	4.726	0.586
9.10	3.124	128 000	5.306	0.648	2.424	0.191	222 900	4.974	0.584
8.07	2.428	128 000	5.254	0.642	3.235	0.364	223 560	5.100	0.612
7.045	1.819	128 260	5.196	0.635	2.981	0.5655	223 560	5.190	0.6235
6.39	1.431	128 260	5.154	0.616	4.945	0.8995	222 900	5.283	0.6365
(i) RUN VI: $A=0.02392$ SQUARE FEET; $D=0.1745$ FEET; AND $\frac{L}{D}=54.44$.									
28.4	31.4	78 600	5.591	0.663	17.56	11.82	83 630	5.408	0.656
26.76	28.1	79 220	5.568	0.667	15.85	9.67	84 260	5.368	0.659
25.8	26.08	80 270	5.558	0.667	13.65	7.15	84 680	5.304	0.657
23.76	21.98	80 690	5.524	0.664	11.84	5.34	84 890	5.244	0.653
21.6	18.02	81 540	5.488	0.661	10.11	3.84	85 310	5.178	0.6475
20.38	15.88	82 160	5.466	0.656	8.51	2.66	85 730	5.105	0.638
18.8	13.6	82 790	5.434	0.658	5.72	1.11	86 150	4.934	0.604

TABLE 2.—(Continued)

Mean velocity, V , in feet per second	Loss of head, H_f , due to friction, in feet	VALUES OF:			Mean velocity, V , in feet per second	Loss of head, H_f , due to friction, in feet	VALUES OF:		
		$\frac{P}{\mu}$, in seconds/feet ²	log R	log (100 f)			$\frac{P}{\mu}$, in seconds/feet ²	log R	log (100 f)
(i) RUN VI: $A = 0.02392$ SQUARE FEET; $D = 0.1745$ FEET; AND $\frac{L}{D} = 54.44$. (Continued)									
1.348	0.0532	81 320	4.282	0.540	14.91	8.66	166 220	5.637	0.6635
1.943	0.0880	83 000	4.450	0.440	13.09	6.61	166 740	5.580	0.660
10.67	4.198	82 370	5.186	0.640	11.68	5.28	167 540	5.533	0.6615
10.0	3.69	82 580	5.158	0.640	10.57	4.255	167 270	5.490	0.654
9.30	3.158	82 580	5.126	0.6345	12.26	5.82	167 000	5.553	0.6615
8.70	2.73	82 370	5.096	0.630	11.0	4.74	168 80	5.509	0.666
7.89	2.233	82 370	5.054	0.628	10.34	4.17	165 700	5.476	0.664
6.945	1.71	82 370	4.999	0.622	9.60	3.59	165 700	5.444	0.663
5.995	1.238	82 580	4.936	0.610	8.80	2.983	166 220	5.407	0.6585
5.243	0.937	82 370	4.876	0.606	7.90	2.378	166 480	5.382	0.654
4.876	0.808	82 370	4.846	0.6045	6.98	1.836	166 740	5.307	0.6495
4.357	0.6445	82 370	4.797	0.604	5.855	1.269	166 740	5.232	0.6415
4.005	0.539	82 370	4.760	0.599	5.03	0.892	166 480	5.186	0.621
3.66	0.445	82 580	4.722	0.594	4.495	0.710	165 960	5.114	0.6195
3.268	0.3465	82 580	4.673	0.584	3.76	0.492	165 700	5.037	0.6145
2.855	0.2603	82 580	4.614	0.577	3.11	0.325	165 960	4.955	0.599
2.402	0.1835	82 790	4.541	0.575	2.595	0.2203	166 480	4.898	0.588
1.86	0.1151	82 790	4.430	0.5955	2.077	0.1428	166 740	4.780	0.592
4.06	0.5235	128 260	4.959	0.574	28.6	32.6	223 920	6.048	0.674
28.35	32.18	127 740	5.800	0.6755	27.3	29.83	223 920	6.028	0.675
26.46	27.78	128 000	5.772	0.670	26.24	27.62	223 920	6.011	0.6755
24.6	23.74	128 520	5.742	0.667	25.36	25.77	223 920	5.996	0.676
23.01	20.8	128 000	5.712	0.6665	24.36	23.93	224 280	5.980	0.6785
21.6	18.35	128 000	5.684	0.668	23.15	21.62	224 280	5.958	0.6795
20.13	15.92	128 260	5.654	0.668	21.95	19.42	224 280	5.934	0.6785
18.23	13.11	128 780	5.612	0.669	20.42	16.92	224 280	5.904	0.6815
16.03	9.94	128 260	5.556	0.660	19.1	14.71	224 100	5.874	0.6785
14.4	7.95	128 000	5.508	0.656	17.6	12.5	224 100	5.838	0.6785
12.23	5.795	128 000	5.437	0.660	15.77	9.90	224 280	5.791	0.673
3.139	0.325	128 260	4.846	0.591	13.84	7.615	224 100	5.734	0.6725
10.89	4.56	126 960	5.353	0.659	12.33	5.995	224 100	5.684	0.668
10.59	4.32	126 700	5.368	0.6595	11.05	4.84	223 920	5.636	0.6705
10.02	3.85	127 480	5.348	0.656	10.6	4.45	224 100	5.618	0.6725
9.395	3.363	128 000	5.322	0.654	9.90	3.886	224 460	5.589	0.6715
8.66	2.838	128 000	5.288	0.6505	9.28	3.395	224 460	5.560	0.6685
7.85	2.318	127 480	5.242	0.648	8.56	2.862	224 460	5.526	0.665
7.085	1.842	127 480	5.198	0.638	7.62	2.252	224 460	5.476	0.662
6.20	1.381	127 480	5.140	0.629	6.66	1.692	224 460	5.417	0.655
5.406	1.028	127 740	5.080	0.6195	5.85	1.30	224 280	5.361	0.652
4.66	0.746	128 000	5.018	0.610	5.05	0.946	224 280	5.297	0.642
2.36	0.1827	128 520	4.724	0.5885	4.37	0.695	224 280	5.234	0.6335
28.53	32.45	167 270	5.921	0.674	3.598	0.464	224 280	5.150	0.627
27.3	29.8	127 270	5.902	0.6745	2.74	0.2535	224 280	5.032	0.602
26.18	27.24	167 000	5.883	0.673	2.103	0.1499	224 280	4.916	0.6025
25.0	24.77	167 000	5.862	0.6705	14.1	7.805	225 360	5.744	0.6665
23.8	22.5	167 000	5.841	0.672	12.75	6.37	222 900	5.696	0.666
21.85	19.05	167 000	5.804	0.6735	11.08	4.85	221 400	5.632	0.670
20.37	16.60	167 270	5.774	0.676	9.94	3.94	222 300	5.5860	0.6735
18.4	13.45	167 000	5.730	0.6725	8.80	3.075	222 900	5.534	0.671
16.85	11.20	166 740	5.690	0.6695	7.98	2.53	223 200	5.493	0.672
14.96	8.795	166 480	5.660	0.668					
(j) RUN VII: $A = 0.02418$ SQUARE FEET; $D = 0.1754$ FEET; AND $\frac{L}{D} = 54.145$.									
10.54	5.956	75 400	5.184	0.803	2.12	0.192	79 850	4.473	0.706
9.63	5.00	75 600	5.107	0.806	2.983	0.4203	80 270	4.624	0.748
8.825	4.22	76 000	5.070	0.806	3.917	0.7375	79 640	4.738	0.756
7.97	3.423	76 200	5.027	0.806	4.31	0.908	79 220	4.778	0.763
7.11	2.597	76 200	4.978	0.7855	24.6	34.2	74 400	5.506	0.827
6.158	1.928	76 600	4.918	0.781	23.46	30.73	74 800	5.470	0.8205
5.476	1.519	76 800	4.868	0.779	22.0	27.04	75 200	5.463	0.822
5.05	1.289	77 000	4.834	0.778	20.43	23.37	75 400	5.432	0.821
4.38	0.9595	77 400	4.774	0.7725	18.57	19.26	75 400	5.390	0.822
3.46	0.5705	77 800	4.674	0.7525	16.71	15.41	75 800	5.347	0.8165
2.648	0.3197	78 000	4.560	0.733	14.8	12.11	76 200	5.297	0.8175
1.783	0.1421	78 200	4.388	0.726	12.85	9.095	76 200	5.235	0.8145
1.347	0.07995	78 400	4.268	0.718	11.26	6.98	76 600	5.180	0.815
1.57	0.1026	79 220	4.339	0.694	9.69	5.095	76 800	5.116	0.810

TABLE 2.—(Continued)

Mean velocity, V , in feet per second	Loss of head, H_f , due to friction, in feet	VALUES OF:			Mean velocity, V , in feet per second	Loss of head, H_f , due to friction, in feet	VALUES OF:		
		$\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{feet}^2}$	$\log R$	$\log$ (100 f)			$\frac{\rho}{\mu}$, in $\frac{\text{seconds}}{\text{feet}^2}$	$\log R$	$\log$ (100 f)
(j) RUN VII: $A = 0.02418$ SQUARE FEET; $D = 0.1754$ FEET; AND $\frac{L}{D} = 54.145$. (Continued)									
24.59	34.61	127 480	5.740	0.833	8.895	4.36	163 880	5.408	0.816
23.24	30.82	127 480	5.720	0.830	7.875	3.38	164 660	5.357	0.8115
21.82	26.82	128 000	5.691	0.8295	6.783	2.505	165 700	5.295	0.812
20.23	23.36	128 000	5.658	0.830	5.76	1.779	166 480	5.227	0.8035
18.69	19.69	127 480	5.622	0.826	5.105	1.371	167 000	5.175	0.796
16.97	16.16	127 740	5.580	0.824	4.295	0.959	166 480	5.099	0.790
15.09	12.64	128 000	5.530	0.820	3.33	0.5605	166 480	4.988	0.778
13.03	9.45	128 000	5.468	0.820	2.369	0.2735	166 480	4.840	0.762
11.7	7.58	128 000	5.420	0.8175	1.697	0.1331	167 000	4.696	0.740
9.93	5.50	128 000	5.354	0.821	2.876	0.4155	167 000	4.925	0.776
10.46	6.05	131 900	5.384	0.818	3.815	0.7465	167 000	5.048	0.784
9.6	5.088	130 600	5.343	0.816	10.1	5.755	222 600	5.596	0.826
8.75	4.195	129 560	5.298	0.812	9.26	4.82	223 200	5.560	0.8245
7.81	3.33	129 040	5.246	0.812	8.41	3.942	223 200	5.518	0.821
6.76	2.441	129 040	5.184	0.802	7.325	2.95	223 200	5.458	0.815
5.76	1.753	128 780	5.115	0.798	6.41	2.238	222 900	5.400	0.8115
4.895	1.257	128 520	5.043	0.7945	5.55	1.675	222 900	5.337	0.8105
4.22	0.8975	128 000	4.977	0.7785	4.96	1.33	222 900	5.288	0.808
2.966	0.429	128 000	4.824	0.763	4.335	1.01	223 200	5.230	0.8045
1.571	0.1076	128 000	4.548	0.714	3.08	0.505	223 560	5.082	0.801
2.321	0.2486	128 000	4.718	0.739	1.633	0.1348	223 560	4.807	0.7795
3.683	0.685	128 780	4.920	0.7775	2.42	0.3003	223 560	4.978	0.7855
24.44	34.62	167 000	5.856	0.830	3.75	0.748	222 600	5.166	0.800
23.22	31.59	167 000	5.833	0.842	24.5	34.97	222 900	5.982	0.840
21.59	27.2	167 000	5.801	0.841	23.62	32.87	222 900	5.966	0.8445
20.02	23.39	167 000	5.768	0.840	22.0	28.73	222 900	5.935	0.849
18.53	19.82	167 000	5.735	0.836	20.4	24.91	222 900	5.902	0.852
17.05	16.60	167 000	5.698	0.8315	18.88	21.13	222 900	5.868	0.848
15.44	13.61	167 000	5.656	0.8315	17.3	17.60	222 900	5.831	0.844
13.52	10.23	167 000	5.598	0.822	15.51	14.02	222 900	5.784	0.841
11.81	7.82	167 000	5.540	0.822	13.83	10.98	222 900	5.734	0.834
10.71	6.357	167 000	5.497	0.818	12.21	8.50	222 900	5.680	0.830
10.36	6.00	168 890	5.488	0.822	10.68	6.46	222 900	5.621	0.827
9.62	5.138	164 400	5.444	0.8185					

